Portfolio Theory

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Investment Analysis &
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Lecture 2

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Investment Process

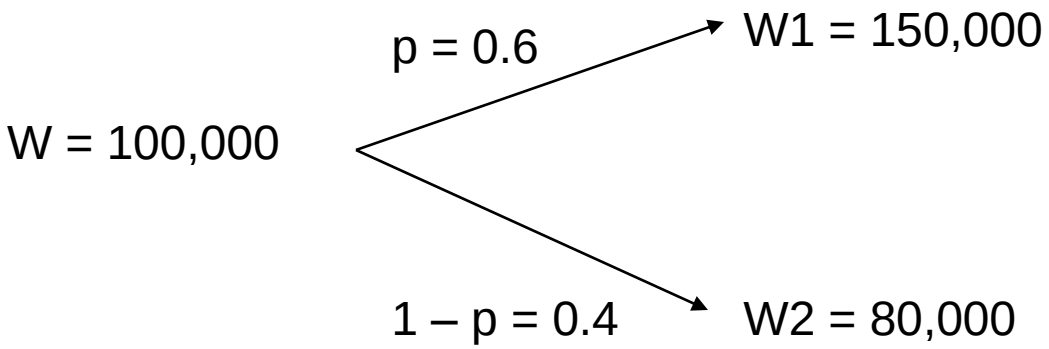
- Investment Process consist of two broad tasks
  1. Security & Market Analysis
     - Assessment of risk and return of the entire set of possible investment vehicles
  2. Portfolio Theory
     - Formation of Optimal Portfolios

Basic Themes of Portfolio Theory

1. Investors avoid risk and demand premium for engaging in risky investments
   - Risk Premium
2. Quantification of investors' personal trade offs between portfolio risk & expected return
   - Utility Function
3. We cannot evaluate risk of an Asset Separate from the Portfolio of which it is a part
   - Impact on the volatility of the entire portfolio

Risk & Risk Aversion

- Presence of Risk mean that more than one outcome is possible



- How will you evaluate this investment?

Risk & Risk Aversion

- Expected Wealth

  $E(W) = p\ W1 + (1- p)\ W2$

- What is the expected profit or loss?

  22,000

- Is this a good investment?

Risk & Risk Aversion

- Variance

  $\sigma^2 = p\,[W1 - E(W)]^2 + (1 - p)\,[W2 - E(W)]^2$

- Standard Deviation

  square root of the variance

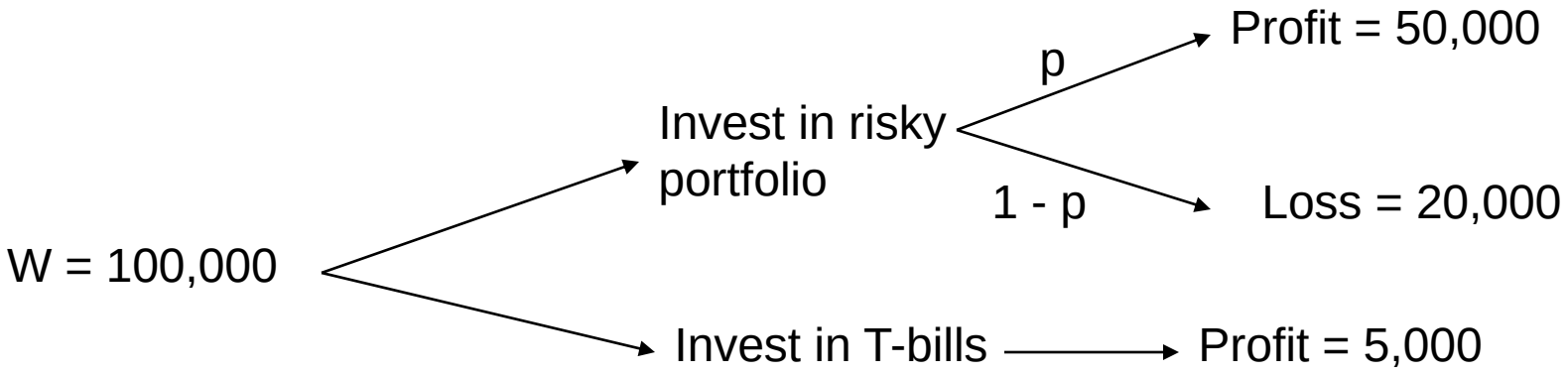
  34,292.86

- Is the expected profit large enough to justify such risk?

Risk & Risk Aversion

Depend on the alternative investment

Suppose the alternate is to invest in Treasury Bills that provide a one year return of 5%



Risk & Risk Aversion

- Risk Premium

  The difference between Expected rate of return and that available on the alternative risk free investments

  risk premium = $E(r) - r_f$

Risk, Speculation & Gambling

- Speculation

  "The assumption of considerable business risk in obtaining commensurate gains"

- Gambling

  "To bet or wager on an uncertain outcome"

OR

the assumption of risk for no purpose but enjoyment of the risk itself

- To convert gamble into speculation requires an adequate risk premium to compensate for the risk they bear
- Heterogeneous Expectations
- Always ask “why is the other willing to invest in the side of a risky prospect that I believe offers a negative expected profit”

Risk Aversion & Utility Functions

- Risk Averse investors are willing to reject investments that are fair game or worse
  - Fair Game – Risk Premium is zero
- Only willing to undertake “risk-free” or speculative prospects with positive risk premia
- Penalize the expected rate of return of a risky prospect by a certain percentage to account for the risk involved

Utility Functions

- Risk Return Profiling
- Assume that each Investor can assign utility to competing investment portfolio based on expected risk and return of those portfolios
- Higher utility => better risk return profile for the selected investor

Utility Function

Commonly employed by financial analysts and AIMR (Association of Investment Management and Research)

$U = E(r) – 0.005A\sigma^2$

where

E(r) is the expected return of the portfolio;

σ^2 is the variance of returns; and

A is an index of investor's risk aversion

- Certainty Equivalent Rate

  the rate that the risk free investment need to offer with certainty to be comparable to the risky portfolio

Mean Variance (M – V) Criterion

- Portfolio A is better than Portfolio B if

1. $E(r_A) \geq E(r_B)$

2. $\sigma_A \leq \sigma_B$

Indifference Curves

Graph of all portfolio that have same utility on mean standard deviation plane

Portfolio Risk

Portfolios are composed of diverse type of assets

Must account for the interplay between asset returns when evaluating the portfolio risk

Hedging

Diversification

Portfolio Mathematics

- Expected Return

  $E(r) = \Sigma P(s)\, r(s)$

  for all scenarios s

- Variance

  $\sigma^2 = \Sigma P(s) [\, r(s) - E(r) \,]^2$

- Rate of return of the portfolio is the weighted average of the rate of return of each asset in the portfolio

Portfolio Mathematics

- When a risky asset is combined with a risk free asset, portfolio standard deviation equals the risky asset standard deviation multiplied by the portfolio proportion invested in the risky asset
- Covariance
- Correlation
- When two risky assets are combined with weights w_1 and w_2

$$\sigma^2_P = w_1^2 \sigma^2_1 + w_2^2 \sigma^2_2 + 2\, w_1\, w_2\, \mathrm{Cov}(r_1, r_2)$$

Capital Allocation Between Risky Asset and Risk Free Asset

Capital Allocation Decision

- Decision to place proportion of overall portfolio in safe risk-free assets
- Generally top down approach
  - Capital decisions generally made at high organizational level
  - Choice of the selection of specific securities within each asset class delegated to portfolio managers

Portfolios of One Risky Asset & One Risk Free Asset

- Suppose you have already decided the composition of the risky portfolio P and the only concern is what proportion of the investment budget be invested in the risky portfolio.
- Let ‘y’ be the proportion invested in P; and let ‘1-y’ be invested in risk free asset F

- Let ‘C’ be the complete portfolio

$$r_C = y\, r_P + (1 - y)\, r_f$$

$$E(r_C) = y\, E(r_P) + (1 - y)\, E(r_F)$$

$$= r_f + y\,(E(r_P) - r_f)$$

but $\sigma_C = y\, \sigma_P$

$$y = \sigma_C / \sigma_P$$

$$E(r_C) = r_f + (\sigma_C / \sigma_P)\,(E(r_P) - r_f)$$